Lattice QCD, O.P.E. and the Standard Model

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December 24, 1999

A number of old and new methods for computing $K \to \pi\pi$ amplitudes on the lattice are reevaluated. They all involve a non-perturbative determination of matching coefficients. I will show how problems related to operator mixing can be greatly reduced by introducing the O.P.E. of hadronic currents directly on the lattice. Applications to the evaluation of CP-symmetric ($\Delta I = 1/2$ rule) and CP-violating (ϵ'/ϵ) processes are presented.

I. Introduction

The Kaon sector of the Standard Model may still have some unexpected surprise in store for us. Uncovering them requires a very good non-perturbative knowledge of hadronic matrix elements. Despite the many non-perturbative approaches developed to this aim (chiral lagrangians, $1/N$-expansions,...) *, it seems that only lattice based computations can cope with this formidable problem.

There are two major difficulties, however, which arise in the calculation of hadronic matrix elements in lattice QCD.

1. Decay amplitudes into two or more particles cannot be directly accessed in Euclidean space, as a consequence of the Maiani and Testa (MT) no-go theorem [6], except at threshold, where, we recall, final state interactions are absent.

2. Operators can mix with operators of lower dimension with coefficients which diverge as inverse powers of the lattice spacing. These contributions must be computed non-perturbatively and subtracted, most often leading to prohibitively large statistical errors [7].

In this talk I will be mainly concerned with the second of these two problems. In Section II I recall some of the old methods proposed to deal with the question of operator mixing in the case of Wilson fermions (either improved or not) †. In the discussion I will

* Excellent reviews can be found in refs. [1]–[5].

make explicit reference to the problem of explaining the large value of the ratio between the $I=0$ and the $I=2$ $K \to \pi\pi$ amplitudes (I is the isospin of the two pions in the final state). In Section III I will illustrate a new approach based on the direct use of the O.P.E. on the lattice [9]. The idea is to measure in Monte Carlo simulations the small x-behaviour ($a \ll |x| \ll \Lambda_{\rm QCD}^{-1}$, a lattice spacing) of the hadronic matrix elements of the product of two hadronic currents and to compare it with the x-behaviour of its O.P.E. In the O.P.E. formula one should look at the Wilson coefficients as known functions (assumed to have been computed in perturbation theory), while the matrix elements of the corresponding operators are treated as fitting parameters. If the condition $a \ll |x| \ll \Lambda_{\rm QCD}^{-1}$ is satisfied, perturbation theory applies, up to corrections vanishing as some power of $a/|x|$, and the best-fit values of the above parameters will automatically supply the physical matrix elements of the finite, renormalized (continuum) operators without the need of knowing their expression in terms of bare lattice operators. A feasibility study of this method in the case of the two-dimensional O(3) σ-model has been successfully carried out in ref. [10]. Applications of the general philosophy underlying this approach to the lattice computation of CP-violating $K \to \pi\pi$ amplitudes, relevant for the evaluation of ϵ'/ϵ [1]–[5], will be presented in Section IV. For lack of space I will not discuss here a recent interesting extension of the above ideas, in which also the Wilson coefficients are extracted in a non-perturbative way from lattice data [11]. In this stronger formulation the O.P.E. method has been already concretely applied by the authors of ref. [11] to the computation of the first moment of D.I.S. structure functions. Conclusions and an outlook on the perspectives of using the lattice O.P.E. method and of its potentialities and limitations can be found in Section V.

II. The old approaches to the $\Delta I = 1/2$ rule

One of the major puzzles remaining in hadronic physics is the origin of the so-called "$\Delta I = 1/2$ rule" in non-leptonic kaon decays. Decays in which isospin changes by $\Delta I = 1/2$ are greatly enhanced over those with $\Delta I = 3/2$. One finds experimentally (neglecting CP violation)

$$\frac{\mathcal{A}(K \to \pi\pi[I=0])}{\mathcal{A}(K \to \pi\pi[I=2])} \approx 20\,. \tag{1}$$

Although the origin of this large enhancement is not theoretically well understood, we do know that, in a QCD-based explanation, most of the enhancement must come from long distance, non-perturbative physics, or else new physics is winking at us here!

† For the use of staggered fermions see ref. [8]. For the new proposals based on Ginsparg-Wilson, overlap or domain wall fermions see the many interesting contributions to these Proceedings.

Let me briefly discuss the source of the difficulties in calculating $\mathcal{A}(K \to \pi\pi)$ in lattice QCD [12] [7].

For scales below M_W, but above the charm quark mass, the $\Delta S = 1$ part of the effective weak Hamiltonian, $\mathcal{H}_{\rm eff}^{\Delta S=1}$, can be written in the form

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \lambda_u \frac{G_F}{\sqrt{2}} \left[C_+(\mu, M_W)\widehat{O}^{(+)}(\mu) + C_-(\mu, M_W)\widehat{O}^{(-)}(\mu) \right] , \quad (2)$$

$$O^{(\pm)} = \left[(\bar{s}\gamma_\mu^L d)(\bar{u}\gamma_\mu^L u) \pm (\bar{s}\gamma_\mu^L u)(\bar{u}\gamma_\mu^L d) \right] - [u \leftrightarrow c] , \quad (3)$$

where $\gamma_\mu^L = \gamma_\mu(1-\gamma_5)/2$, $\lambda_u = V_{ud}V_{us}^*$, G_F is the Fermi constant and μ is the subtraction point. Throughout this paper I will denote finite renormalized operators with a $\hat{}$ on top.

The operators $\widehat{O}^{(+)}$ and $\widehat{O}^{(-)}$ have different transformation properties under isospin. In particular, $\widehat{O}^{(-)}$ is pure $I = 1/2$, whereas $\widehat{O}^{(+)}$ contains parts having both $I = 1/2$ and $I = 3/2$. An explanation of the $\Delta I = 1/2$ rule thus requires that the $K \to \pi\pi$ matrix element of $C_-\widehat{O}^{(-)}$ be substantially enhanced compared to that of $C_+\widehat{O}^{(+)}$.

Part of the enhancement is provided by the ratio of Wilson coefficients, C_-/C_+, in their renormalization group evolution from $\mu \sim M_W$ down to $\mu \sim 2$ GeV, where one finds $|C_-/C_+| \approx 2$. This factor is, however, too small by an order of magnitude to explain the $\Delta I = 1/2$ rule. The remainder of the enhancement must come from the matrix elements of the operators, and these are the quantities that we wish to evaluate on the lattice. Attempts in this direction date back to the works of ref. [12]

There is a long list of methods proposed in the literature to deal with the problem of computing on the lattice the hadronic matrix elements that enter in the non-leptonic weak decay amplitude. They are all aimed at bypassing the two major difficulties mentioned in the Introduction. In this section I wish to briefly highlight the merits and the drawbacks of the most promising among them. Before doing that let me start by recalling what is the mixing pattern of the operators $O^{(\pm)}$.

II-1. The mixing of $O^{(\pm)}$ on the lattice

As is well known, the explicit breaking of chiral symmetry due to the presence of the Wilson term in the lattice action induces the mixing of the bare operators $O^{(\pm)}$ with operators belonging to different chiral representations. Taking into account the symmetries left unbroken by the lattice regularization, it can be shown that, in terms of bare operators, the renormalized, finite lattice operators possessing the correct chiral transformation properties have the form [13]

$$\widehat{O}^{(\pm)} = \widehat{O}^{(\pm)}_{PC} + \widehat{O}^{(\pm)}_{PV} \quad (4)$$

with

$$
\begin{aligned}
\widehat{O}^{(\pm)}_{PC} &= Z^{(\pm)}_{PC}[O^{(\pm)}_{PC} + \textstyle\sum_{i=1}^{4} C^{(\pm)}_{iPC} O^{(\pm)}_{iPC} + C^{(\pm)}_{\sigma F}\bar{s}\sigma_{\mu\nu}F_{\mu\nu}d + C^{(\pm)}_{\bar{s}d}\bar{s}d] \\
\widehat{O}^{(\pm)}_{PV} &= Z^{(\pm)}_{PV}[O^{(\pm)}_{PV} + C^{(\pm)}_{\sigma\widetilde{F}}\bar{s}\sigma_{\mu\nu}\widetilde{F}_{\mu\nu}d + C^{(\pm)}_{\bar{s}\gamma_5 d}\bar{s}\gamma_5 d]\,.
\end{aligned}
\tag{5}
$$

$O^{(\pm)}_{PC}$ and $O^{(\pm)}_{PV}$ are respectively the parity-conserving (PC) and the parity-violating (PV) parts of the operators $O^{(\pm)}$. The $O^{(\pm)}_{iPC}$'s (i=1,...,4) are (6 dimensional) four-quark operators whose explicit expression can be found in [14]. As we have indicated above, parity-conserving and parity-violating operators renormalize separately, because strong interactions conserve parity.

The mixing pattern shown in eqs. (5) is the same both with the standard Wilson action and with the "tree-level" clover-improved action [15], provided in the latter case quark fields are appropriately improved ("rotated"), according to $q \to (1 + am_0/2)q$, $\bar{q} \to (1 + am_0/2)\bar{q}$, with m_0 the bare quark mass parameter entering in the fermion action. If the action is improved non-perturbatively, i.e. beyond tree-level [16], the form of eqs. (5) will become much more complicated because of the presence of many more operators, needed to correct all higher orders in a, beyond the leading $ag_0^{2n}\log^n a$ terms already taken care of by the tree-level Pauli term in the clover action. To my knowledge a complete analysis of the mixing in this case is still lacking.

Let us now examine in turn the various terms in eqs. (5).

• <u>Dimension 6 operators</u>

Spin and color structure of the operators of dimension 6 contributing here is the same as that of the $\Delta S = 2$ case discussed in ref. [14]. Only the flavor structure is different. Notice that there is no mixing with dimension 6 operators in the parity-violating part.

• <u>Dimension 5 operators</u>

Thanks to the GIM cancellation mechanism, the coefficients $C^{(\pm)}_{\sigma F}$ and $C^{(\pm)}_{\sigma\widetilde{F}}$ are actually finite, because the potential $1/a$ divergence in the matrix elements of $O^{(\pm)}_{PC}$ and $O^{(\pm)}_{PV}$ gets replaced by a $m_c - m_u$ factor. Furthermore the CPS symmetry [17] (CPS = CP $\times$ symmetry under $s \to d$ exchange) makes $C^{(\pm)}_{\sigma\widetilde{F}}$ proportional to $m_s - m_d$ and thus vanishing in the limit of exact vector flavor symmetry (exact $SU(N_f)_V$).

• <u>Dimension 3 operators</u>

The GIM mechanism softens the a priori possible $1/a^3$ divergence of $C^{(\pm)}_{\bar{s}d}$ and $C^{(\pm)}_{\bar{s}\gamma_5 d}$, because, as I said, it has the effect of substituting one factor $1/a$ with $m_c - m_u$. In the case of $C^{(\pm)}_{\bar{s}\gamma_5 d}$ an extra $1/a$ power is taken care of by CPS symmetry which has the effect of replacing it by a $m_s - m_d$ factor.

II-2. CP-conserving $K \to \pi\pi$ amplitudes

In this section I wish to briefly describe a number of methods that have been proposed to deal with the problem of computing CP-conserving $K \to \pi\pi$ amplitudes on the lattice.

1. In ref. [17] it was suggested to work with $m_s = m_d$ and calculate the amplitudes

$$\mathcal{A}^{(\pm)} = \langle \pi(\vec{p}_1 = 0)\pi(\vec{p}_2 = 0)|\widehat{O}^{(\pm)}(\mu)|K(\vec{p}_K = 0)\rangle\Big|_{m_s = m_d} \tag{6}$$

with all three particles at rest. We have seen that, setting $m_s = m_d$, GIM cancellation and CPS-symmetry cause all mixings in the parity-violating part to vanish, removing the need for subtractions altogether. Since one is working at threshold with the two pions at rest, the MT no-go theorem [6] does not apply. So both difficulties mentioned in the Introduction are overcome with this simple choice. The method requires, however, a large extrapolation from the unphysical, off-shell point, $m_s = m_d$, to the physical one with the use of chiral perturbation theory [19]. This procedure can be particularly delicate if lattice numbers are extracted from quenched simulations, due to the presence of chiral logarithms [20].

2. An alternative method [9] consists in working with the non-perturbatively $O(a)$ improved clover action (for which there are no errors of $O(a)$ in the spectrum [15] and on-shell matrix elements of improved currents obey the continuum chiral Ward identities up to $O(a^2)$ [16]). Choosing quark masses such that $m_K = 2m_\pi$, one measures the $K \to \pi\pi$ amplitudes again with all particles at rest. Since in these kinematical conditions, the total momentum transfer vanishes ($\Delta p = 0$), one can prove that the matrix element of the dangerous, power divergent, subtraction of the operator $\bar{s}\gamma_5 d$ is now of $O(a)$ rather than $O(1)$ and vanishes in the continuum limit. Furthermore pions are at rest and again the MT no-go theorem does not apply. Besides the need of a (perhaps less severe) chiral extrapolation (unlike the previous case here we are dealing with on-shell amplitudes), a problem with this method is the difficulty of tuning quark masses with a sufficiently high accuracy to have the condition $m_K = 2m_\pi$ satisfied. This problem can be, however, alleviated by explicitly performing the subtraction of the $\bar{s}\gamma_5 d$ operator.

3. The old proposal of ref. [22] is based on the evaluation of the $K \to \pi$ matrix elements of (the positive parity part of) the weak Hamiltonian and makes use of chiral perturbation theory, in the form of Soft Pion Theorems (SPT's), to connect $K \to \pi$ to $K \to \pi\pi$ amplitudes. Since only single-particle states are involved, there are no problems with the MT no-go theorem. The disadvantage of the method is that, as discussed before, the operator mixing problem for the positive parity part of $O^{(\pm)}$ is much more severe than for their negative parity part. This makes an accurate evaluation of the matrix elements of the renormalized operators very difficult.

The relation between $K \to \pi\pi$ and $K \to \pi$ amplitudes can be found from the classical works of refs. [21]. At leading order in chiral perturbation theory the physical amplitude takes the form (for $\Delta p = 0$)

$$\langle \pi^+\pi^- | \widehat{O}^{(\pm)}(\mu) | K^0 \rangle = i\,\gamma^{(\pm)}\,\frac{m_K^2 - m_\pi^2}{f_\pi}\,. \tag{7}$$

As the coefficients $\gamma^{(\pm)}$ appear also in the expression for the $K \to \pi$ matrix element

$$\langle \pi^+(p) | \widehat{O}^{(\pm)}(\mu) | K^+(q) \rangle = -\delta^{(\pm)}\,\frac{m_K^2}{f_\pi^2} + \gamma^{(\pm)}\, p\cdot q\,, \tag{8}$$

by studying this matrix element on the lattice as a function of $p \cdot q$, one can, in principle, determine $\gamma^{(\pm)}$, from which one then obtains the $K \to \pi\pi$ amplitudes [‡].

In order to construct the finite renormalized lattice operators $\widehat{O}^{(\pm)}$ it was suggested in reference [22] to use perturbation theory to determine the finite mixing coefficients ($C_{iPC}^{(\pm)}$ and $C_{\sigma F}^{(\pm)}$) and subtract non-perturbatively the operator $\bar{s}d$, as its mixing coefficient, $C_{\bar{s}d}^{(\pm)}$ (see the first of eqs. (5)), is quadratically divergent. This approach has been tried in ref. [7] with no success as numerical and statistical errors coming from the power divergent subtraction completely obscure the physical signal.

III. Lattice O.P.E.

In this section I want to illustrate a strategy which, in principle, avoids all the difficulties caused by mixing with lower dimension operators and automatically gives the physical matrix elements of the effective weak Hamiltonian with the correct normalization [9].

The method is based on the idea of studying at short distances ($a \ll |x| \ll \Lambda_{\rm QCD}^{-1}$) the x-behaviour of the O.P.E. of two hadronic currents on the lattice. It does not use chiral perturbation theory, and thus in principle applies equally well to the $\Delta S = 1$, $\Delta C = 1$ and $\Delta B = 1$ parts of the weak Hamiltonian. In addition, it allows one to construct an improved weak Hamiltonian (i.e. one having errors of $O(a^2)$), if the improved version of the weak hadronic currents [16] is used. The approach is speculative in the sense that it is likely to require more computational power than is presently available, although it may become practical with the advent of Teraflop machines.

‡ For completeness I recall that, in the same notations used in eqs. (7) and (8), one has $\langle 0 | \widehat{O}^{(\pm)} | K^0 \rangle = i(m_K^2 - m_\pi^2)\delta^{(\pm)}$.

I recall that the standard construction of the non-leptonic weak Hamiltonian begins with the formula

$$\mathcal{H}^{W}_{\rm eff} = \frac{g^2_{W}}{8} \int d^4x \, D_{W}(x; M_{W}) T\Big(J_{\rho L}(x) J^{\dagger}_{\rho L}(0)\Big) \, , \tag{9}$$

where

$$D_{W}(x; M_{W}) = \int \frac{d^4p}{(2\pi)^4} \frac{{\rm e}^{ipx}}{p^2 + M^2_{W}} \tag{10}$$

is the longitudinal part of the W-boson propagator and $J_{\rho L}$ is the (left-handed) hadronic weak current. One then introduces the Wilson operator product expansion in the r.h.s of eq. (9), a step which is justified by the observation that the dominant contribution to the integral comes from very small distances, $|x| \ll M^{-1}_{W}$. For physical amplitudes, one obtains in this way

$$\langle h|\mathcal{H}^{W}_{\rm eff}|h'\rangle = \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu, M_{W}) M^{6-d_i}_{W} \langle h|\widehat{O}^{(i)}(\mu)|h'\rangle \ , \tag{11}$$

where $|h\rangle$ $|h'\rangle$ are hadronic states, d_i is the dimension of the operator $\widehat{O}^{(i)}(\mu)$ and $G_F/\sqrt{2} = g^2_{W}/8M^2_{W}$. The functions $C_i(\mu, M_{W})$ result from the integration of the Wilson expansion coefficients, $c_i(x;\mu)$ (defined in eq. (13) below), with the W-propagator

$$C_i(\mu, M_{W}) M^{6-d_i}_{W} = \int d^4x \, D_{W}(x; M_{W}) c_i(x; \mu) \ . \tag{12}$$

The $\widehat{O}^{(i)}(\mu)$'s are quark and/or gluon operators renormalized at the subtraction point μ. The functions $C_i(\mu, M_{W})$ are evaluated in perturbation theory and their running with μ is dictated by the renormalization group equation which follows from the μ-independence of the l.h.s. of eq. (11). The sum in the expansion (11) is over operators of increasing dimension. As the operator dimension of $\mathcal{H}^{\mathcal{W}}_{\rm eff}$ is 6, we will have to consider in the following only operators with dimensions $d_i \leq 6$, since the contribution from operators with $d_i > 6$ is suppressed by powers of $1/M_{W}$.

All the intricacies and complications of operator mixing in the definition of the finite and renormalized operators, $\widehat{O}^{(i)}(\mu)$, come about because the integrals in (9) and (12) are extended down to the region of extremely small $|x|$. The complicated mixing pattern of the $\widehat{O}^{(i)}(\mu)$'s in terms of bare operators arises from contact terms when the separation of the two currents goes to zero, i.e. when $|x|$ is of the order of a. This observation suggests that a simple way to circumvent these difficulties is to directly determine the matrix elements of renormalized operators by enforcing the validity of the O.P.E. on the lattice for distances $|x|$ much larger than the lattice spacing a, but much smaller than $\Lambda_{\rm QCD}^{-1}$ (and inverse quark masses), i.e. in a region where perturbation theory determines the form of the Wilson expansion.

We should imagine proceeding in the following way. If $J_{\rho L}$ is the appropriately renormalized (and possibly improved) finite lattice current operator, one starts by measuring in a Monte Carlo simulation the hadronic matrix element $\langle h|T(J_{\rho L}(x)J^{\dagger}_{\rho L}(0))|h'\rangle$, as a function of x in the region $a \ll |x| \ll \Lambda_{\rm QCD}^{-1}$. The numbers $\langle h|\widehat{O}^{(i)}(\mu)|h'\rangle$ (eq. (11)) are extracted by fitting the x-behaviour of $\langle h|T(J_{\rho L}(x)J^{\dagger}_{\rho L}(0))|h'\rangle$ to the O.P.E. formula

$$\langle h|T\Big(J_{\rho L}(x)J^{\dagger}_{\rho L}(0)\Big)|h'\rangle = \sum_i c_i(x;\mu)\langle h|\widehat{O}^{(i)}(\mu)|h'\rangle\,, \tag{13}$$

where the Wilson coefficients $c_i(x;\mu)$ are determined by continuum perturbation theory using any renormalization scheme we like. The scale μ should be chosen so that also the inequalities $a \ll 1/\mu \ll \Lambda_{\rm QCD}^{-1}$ are obeyed. Since we only consider operators of dimension 6 or lower, the lattice T-product differs from the right-hand side of eq. (13) by terms of $O(|x|^2\Lambda_{\rm QCD}^2)$, which is then an estimate of the size of the systematic errors intrinsic in this procedure. As a last step we insert the fitted numbers $\langle h|\widehat{O}^{(i)}(\mu)|h'\rangle$ in (11), obtaining in this way an explicit expression for the matrix elements of $\mathcal{H}^W_{\rm eff}$.

The procedure illustrated above requires the existence of a window, $a \ll |x| \ll \Lambda_{\rm QCD}^{-1}$, in which the distance between the two currents is sufficiently small that perturbation theory can be trusted, but large enough that lattice artifacts, which are suppressed by powers of $a/|x|$, are tiny. For such a window to exist we need to have an adequately small lattice spacing. At the same time the physical volume of the lattice must be sufficiently large to allow the formation of hadrons.

A few remarks may be useful at this point.

• The method determines directly the "physical" matrix elements of the operators appearing in the O.P.E. of the two currents, i.e. the matrix elements of the finite, renormalized operators $\widehat{O}^{(i)}(\mu)$, without any reference to the magnitude of the W-mass. This means that it will not be necessary to probe distances of $O(1/M_W)$ with lattice calculations.

• Since it is the continuum O.P.E. which determines the operators appearing in the lattice expansion (11), these are restricted by the continuum symmetries. For $\Lambda_{\rm QCD}^{-1} \gg |x| \gg a$, the lattice O.P.E. matches, in fact, that of the continuum with discretization errors suppressed by powers of $a/|x|$.

• Unlike the methods discussed before, this approach automatically yields hadronic amplitudes that are properly normalized (in the renormalization scheme in which the Wilson coefficients appearing in eq. (13) are computed).

As for the applicability of this strategy to the actual case of the CP-conserving $\Delta S = 1$ processes, fortunately in the expansion (11) there appear no operators of dimension lower than 6. If lower dimension operators were present they would dominate at short distances, since their Wilson coefficients would diverge as powers of $1/x$ (up to logarithmic corrections). In this situation it would be virtually impossible to pick out the matrix elements of the interesting dimension 6 operators.

Operators of dimension 6 have Wilson coefficients which vary logarithmically with $|x|$. At leading order their expression is of the form

$$c_i(x;\mu) \propto \left(\frac{\alpha_s(1/|x|)}{\alpha_s(\mu)}\right)^{\frac{\gamma_0^{(i)}}{2\beta_0}} = 1 + \frac{\alpha_s}{4\pi}\gamma_0^{(i)} \log(|x|\mu) + \dots\,, \tag{14}$$

where $\gamma_0^{(i)}$ is the one-loop anomalous dimension of the operator $O^{(i)}$ and β_0 is the coefficient of the one-loop term in the β-function.

In the case of the $\Delta S = 1$ part of $\mathcal{H}_{\rm eff}^{W}$, the operators which can appear in (11) are $\widehat{O}^{(\pm)}$ (eq. (3)) and in addition

$$O' = (m_c^2 - m_u^2)\, \bar{s}(\overrightarrow{D_\mu} - \overleftarrow{D_\mu})\gamma_\mu^L d\,. \tag{15}$$

The mass factor in the r.h.s. of eq. (15) comes from a combination of the GIM mechanism, which causes O' to vanish when $m_c = m_u$, and chiral symmetry, which requires both quarks to be left-handed, leading to a GIM factor quadratic in the quark masses. Since O' has dimension 6, its coefficient function depends only logarithmically on $|x|$.

The anomalous dimensions of the three relevant operators ($\widehat{O}^{(\pm)}$ and O') are

$$\gamma_0^{(+)} = 4, \qquad \gamma_0^{(-)} = -8, \qquad \gamma_0' = 16\ . \tag{16}$$

Actually, the contribution of O' to the r.h.s. of eq. (13) can be determined separately (since its matrix elements do not require any subtraction) and needs not be fitted. The anomalous dimensions of the operators $O^{(\pm)}$ are well separated from one another, so it might be possible to determine the amplitudes $\langle h|\widehat{O}^{(\pm)}(\mu)|h'\rangle$ and obtain the physical matrix elements of $\mathcal{H}_{\rm eff}^{\Delta S=1}$.

IV. CP-violating $\Delta S = 1$ processes

For CP-violating processes in kaon decays (or for B decays), where top-penguin diagrams enter at a Cabibbo-allowed level, the strategies described at the points 1. and 2. of sec. II-2 for the negative parity operators fail, because the GIM mechanism is not operative, as the top quark is too heavy to let it propagate on the lattice. In particular

$O^{(\pm)}$ mix with penguin operators (see below). This makes the calculation of the mixing matrix of comparable difficulty to that for the positive parity operators discussed at point 3. of sec. II-2. For positive parity operators, the analysis carried out in sec. II-1 applies with the difference that the mixing coefficients of the color magnetic operator and scalar density become more divergent [22] and this will make the numerical determination of the renormalized operators less precise. In order to circumvent these problems we propose two methods involving a fictitious top quark (with mass $\widetilde{m}_t$) which is momentarily taken to be light enough to propagate on the lattice.

IV-1. The Renormalization Group method

The basic idea of what we may call the "Renormalization Group method" is to work with two different scales: the first, μ, is larger than $\widetilde{m}_t$, so that the corresponding operator basis is as in the previous sections; the second, μ', is smaller than $\widetilde{m}_t$ so that a full set of new (penguin) operators is generated. The matrix elements of the operators renormalized at the scale μ are computed numerically following one of the strategies explained in sec. II. By matching the result to the amplitude expressed in terms of operators renormalized at μ', we extract their matrix elements. In this way, at least in principle, we can obtain the matrix elements of the penguin operators without directly computing them. Let me now present the details of this procedure.

At scales $a^{-1} \gg M_W \gg \mu \gg \widetilde{m}_t$, where the fake top quark is active, the CP-violating part of the $\Delta S = 1$ part of the effective weak hamiltonian, $\mathcal{H}_{\rm eff}^{\Delta S=1}|_{\overline{CP}}$, takes the form (see eq. (2))

$$\mathcal{H}_{\rm eff}^{\Delta S=1}|_{\overline{CP}} = \lambda_t \frac{G_F}{\sqrt{2}}[C_1(\mu, M_W)(\widehat{O}_1^t(\mu) - \widehat{O}_1^c(\mu)) + +C_2(\mu, M_W)(\widehat{O}_2^t(\mu) - \widehat{O}_2^c(\mu))]\,, \tag{17}$$

$$O_1^q = (\bar{s}\gamma_\mu^L d)(\bar{q}\gamma_\mu^L q) \quad q = u, c, t \tag{18}$$

$$O_2^q = (\bar{s}\gamma_\mu^L q)(\bar{q}\gamma_\mu^L d) \quad q = u, c, t \tag{19}$$

where $\lambda_t = V_{td}V_{ts}^*$. The operators $\widehat{O}^{(\pm)}$ of eq (3) and the corresponding Wilson coefficients, $C_\pm$, in eq (2) can be immediately written as linear combinations of the operators and coefficients above.

At scales μ' below $\widetilde{m}_t$, when GIM is not operative, the form of the effective weak Hamiltonian becomes

$$\mathcal{H}_{\rm eff}^{\Delta S=1}|_{\overline{CP}} \;=\; \frac{G_F}{\sqrt{2}} \sum_{j=1}^{N} C_j(\mu', M_W, \widetilde{m}_t)\widehat{O}_j(\mu') \tag{20}$$

where now many new operators appear with a complicated mixing pattern with lower dimensional operators [§]. In fact in the absence of GIM cancellation, in order to write a renormalized lattice version of each of the operators $\widehat{O}_j$ it is required to subtract lower dimensional operators with appropriate (power divergent) coefficients, and account for the mixing with 6 dimensional operators. This is true for both positive and negative parity sectors. The coefficient functions appearing in eq. (20) have been calculated up to non-leading order in perturbation theory in refs. [24]–[29].

In order to avoid the difficulties mentioned above we have to keep the GIM mechanism operative and this is done by having on the lattice a fictitious top quark with mass satisfying

$$1/a \gg \widetilde{m}_t \gg m_c \gg \Lambda_{\mathrm{QCD}} \,. \tag{21}$$

To illustrate how the method works in practice let me restrict the discussion below to the simpler case of the negative parity operators. One begins by evaluating on the lattice at a renormalization scale satisfying $\mu \gg \widetilde{m}_t$ the two matrix elements

$$\mathcal{M}_i(\mu, \widetilde{m}_t) \equiv \langle h|\widehat{O}^c_i(\mu) - \widehat{O}^t_i(\mu)|h'\rangle \,, \quad i = 1, 2 \tag{22}$$

Since GIM is operative, the analysis of point 2. in sec. II-2 applies (with $m_u \to m_t$) and we can define $\widehat{O}^c_i(\mu) - \widehat{O}^t_i(\mu)$ in terms of bare lattice operators (by appropriately subtracting the operator $\bar{s}\gamma_5 d$, as discussed in ref. [9]).

On the other hand, we can also consider the same matrix elements for a renormalization scale $\widetilde{m}_t \gg \mu' \gg m_c$. Since in this case the GIM mechanism is not at work, all the operators which appear in eq. (20) can contribute and the amplitude $\mathcal{M}_i(\mu, \widetilde{m}_t)$ will take the form

$$\mathcal{M}_i(\mu, \widetilde{m}_t) = \sum_{j=1}^{N} \widehat{Z}_{ij}(\mu', \mu, \widetilde{m}_t)\langle h|\widehat{O}_j(\mu')|h'\rangle \,. \tag{23}$$

The rectangular matrix $\widehat{Z}_{ij}$ can be calculated perturbatively by matching the theory with ($\mu \gg \widetilde{m}_t$) and without ($\mu' \ll \widetilde{m}_t$) the fictitious top quark [¶]. The results for $\widehat{Z}$ at non-leading order can be reconstructed from the works of refs. [24]–[29]. Notice that in absence of QCD corrections one would have $C_1 = 0$, $C_2 = 1$.

[§] A convenient basis of operators when QCD corrections are taken into account can be found in the papers quoted in ref. [23]. See also refs. [1] [2].

[¶] This is exactly the method used to calculate the Wilson coefficients in the theory with the physical top quark mass, except that in the physical case one must simultaneously integrate out both W boson and top quark. Here we are effectively integrating out the W first and the fictitious top quark afterwards.

We now choose a fixed value of μ' and let $\widetilde{m}_t$ vary. The matrix elements of interest, $\langle h|\widehat{O}_j(\mu')|h'\rangle$, are obtained by fitting the $\widetilde{m}_t$ dependence of the r.h.s. of eq. (23) to $\mathcal{M}_i(\mu,\widetilde{m}_t)$, computed numerically as in eq. (22), and using the renormalization matrix $\widehat{Z}_{ij}$ calculated perturbatively. Since the dependence on $\widetilde{m}_t$ is only logarithmic, this will not be an easy job. The procedure is analogous to our use of the x-dependence in sec. III to find the renormalized matrix elements of operators appearing in the weak Hamiltonian. Having determined the numbers $\langle h|\widehat{O}_j(\mu')|h'\rangle$, we can insert them into the expression (20) for the effective Hamiltonian. At this point the constraint $M_W \gg \widetilde{m}_t$ can be removed since the Wilson coefficients of the operators appearing in (17) can be computed perturbatively for arbitrary values of $\widetilde{m}_t$, including $\widetilde{m}_t = m_t$.

As for the errors involved in this procedure, it should be observed that for an accurate determination of the matrix elements of $\widehat{O}_i(\mu)$ (or of $\widehat{O}_j(\mu')$) the condition $\mu \gg \widetilde{m}_t \gg \mu' \gg \Lambda_{\rm QCD}$ is not enough. It must also be required that the typical scale, $\Lambda_{hh'}$, of masses and external momenta appearing in the physical process $h' \to h$ be much smaller than $\widetilde{m}_t$. This is because in the matching procedure terms of $O(\Lambda_{hh'}/\widetilde{m}_t)$ are neglected.

IV-2. The O.P.E. method

An alternative method, in the same spirit of the approach followed in sec. III, is the following. We can avoid the need for any non-perturbative subtraction by separating the two currents in x space and having a fictitious propagating top quark. Thus, as in sec. III, we directly match the x-behaviour of $\langle h|T(J_{\rho L}(x)J^\dagger_{\rho L}(0))|h'\rangle^{\Delta S=1}_{\rm top}$ (the subscript indicates the presence of the fictitious top) to the formula

$$\langle h|T(J_{\rho L}(x)J^\dagger_{\rho L}(0))|h'\rangle^{\Delta S=1}_{\rm top} = \sum_j c_j(x;\mu',\widetilde{m}_t)\langle h|\widehat{O}^{(j)}(\mu')|h'\rangle\,, \tag{24}$$

where

$$J_{\rho L}(x)J^\dagger_{\rho L}(0)|^{\Delta S=1} = \bar{s}(x)\gamma^L_\mu t(x)\bar{t}(0)\gamma^L_\mu d(0) - \bar{s}(x)\gamma^L_\mu c(x)\bar{c}(0)\gamma^L_\mu d(0)\,. \tag{25}$$

The coefficients $c_{1,2}(x;\mu',\widetilde{m}_t) \equiv c_{1,2}(x;\mu')$ are the same as those of sec. II. The others are complicated functions of the anomalous dimension matrix which can be worked out from the results of refs. [26] and [28] and computed numerically.

Both methods proposed in this section require small enough lattice spacings to accommodate a number of scales. Like the method of sec. III their full implementation is likely to require the next generation of supercomputers.

V. Conclusions

The new experimental results on ϵ'/ϵ [30], combined with the old ones [31], have prompted a wage of theoretical work aimed at testing the validity of the Standard Model in this crucial corner of the theory. Lattice, as shown recently by the investigation carried out in ref. [32], has the potentiality of providing a clearcut answer to the question on whether CP-violation can be understood and described within the framework of the Standard Model, as it is formulated today. In my opinion all possible efforts should be addressed by the lattice community to this fundamental issue.

With this in mind in this talk I have reviewed a number of old and new approaches aimed at studying the $\Delta I = 1/2$ rule on the lattice, using Wilson-like fermions (similar methods could also be used for staggered fermions [8]) and I have discussed a new approach which can be equally well applied to CP-conserving and CP-violating $\Delta S = 1$ processes.

In particular the methods of refs. [17] and [22] involving $K \to \pi\pi$ and $K \to \pi$ amplitudes, respectively have been reevaluated. The last approach is likely to be more difficult than the first one, because of the large number of mixing coefficients which have to be determined non-perturbatively. It may however give complementary information to the results obtained with the $K \to \pi\pi$ methods (points 1. and 2. of sec. II-2) and provide a check of the accuracy of chiral extrapolations.

To overcome the difficulties inherent in the construction of finite renormalized lattice operators, a new strategy, based on the study of the short distance behaviour of the O.P.E. of two hadronic weak currents on the lattice, was proposed in ref. [9]. The approach is theoretically very appealing and can be applied to both CP-conserving and CP-violating $\Delta S = 1$ processes. There are also good indications [11] that the approach can be extended in a way as to allow a fully non-perturbative evaluation of the (first few) moments of D.I.S. structure functions [‖].

An other interesting feasibility study in the direction of testing the O.P.E. method in actual simulations has been undertaken in ref. [10]. It consists in the study of the small x-behaviour of the O.P.E. in the two-dimensional lattice O(3) σ-model. A preliminary analysis of Monte Carlo data indicates that the measured small x-behaviour of the one-particle matrix elements of products of operators matches the logarithmic behaviour expected from perturbative calculations, thus allowing the (non-perturbative) evaluation of the matrix elements of the operators appearing in the O.P.E.

The advent of Teraflop Supercomputers may render the O.P.E. method a viable strategy also for QCD, allowing us to directly extract physical amplitudes from Monte

[‖] The generalization proposed in ref. [11] makes use of (gauge non-invariant) quark and gluon states in order to get a sufficiently large set of equations and be able to extract all the unknown coefficients and matrix elements. In these circumstances non-BRST operators vanishing by the equations of motion can contribute to the O.P.E. [33]. If they are not included, care must be exerced in choosing the external states in order to avoid contaminations from these unwanted operators onto the relevant matrix elements.

Carlo data, which may compete with the more standard approaches illustrated in sec. II or with those, based on Ginsparg-Wilson, overlap or domain wall fermions [34], that are now starting to be developed.

Acknowledgements

I would like to thank the Organizers of "CHIRAL '99", and especially Prof. Ting-Wai Chiu, for the warm hospitality and for the very exciting scientific atmosphere that they have been able to create during the meeting.

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